Optimierung sicherheitskritischer IFR Helikopter-Trajektorien in Alpinem Gelände mit MILP

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Visual vs. Instrument Meteorological Conditions (VMC vs. IMC)

https://www.youtube.com/watch?app=desktop&v=fwAotwl8wQ4
Flying helicopters nowadays

Flight simulator, REGA, AW 109 SP, Training site, Kloten
Current situation
Goals

- 3D trajectory for Instrument Meteorological Conditions

- Minimize flight time
- Minimize height above terrain

- Suitable terrain representation
- Optimization approach
- Constraint formulation
Collision Probabilities

**Collision probability**: probability of a position, given by a GNSS sensor (e.g. $\sigma_x = \sigma_y = 200$ m, $\sigma_z = 300$ m), to be in the terrain.

https://doi.org/10.33012/2019.16909
Iso-probable point cloud
The iso-probable surface is approximated using the Incremental Delaunay Triangulation algorithm by Sakhi (2020)
Triangulation Error

Maximum error 20m

# triangles: 766

Maximum error 300m

# triangles: 49
Mixed Integer Linear Programming (MILP)

Simple Example
MILP – Basic Model

- **Input**
  - Iso-probable surface (TIN)
  - Coordinates (Start & Finish)

- **Assumption**
  - Discrete time steps

- **Output**
  - Waypoint coordinates
  - Speed per waypoint
MILP – Terrain Avoidance

\[ z \geq h \]

\[ b_t \in \{0,1\} \quad \forall t \]

\[ \sum_t b_t = 1 \quad \text{activation para} \]

\[ x = \sum_j \lambda_j x_j \]

\[ y = \sum_j \lambda_j y_j \]

\[ h = \sum_j \lambda_j z_j \]

\[ \sum_j \lambda_j = 1 \quad 0 \leq \lambda_j \leq 1 \quad \forall j \]
**MILP – Flight Dynamics**

**Horizontal approximation**

- **# directions**: 36
- **Step size**: 10°

**Vertical approximation**

- **Step size**: 1°
- **Horizontal**: 0°
- **Climb**: 11° to 24°
- **Descent**: -3° to -9°

Approximation of horizontal directions.
(Example with 8 directions)

\[
\begin{align*}
    x_{i+1} &= x_i + v_x \\
    y_{i+1} &= y_i + v_y \\
    z_{i+1} &= z_i + v_z
\end{align*}
\]

\[v_x, v_y, v_z: \text{depend on flight phase}\]

\[
dir_{h,i+1} - dir_{h,i} \leq \pm 10°
\]
MILP – Objective

- \( \text{minimize } \sum_{t=1}^{T} \alpha \cdot (t \cdot \delta) + (z_t - h_t) \)

\( T: \text{maximum time steps} \)
\( \alpha: \text{weight factor} \)
\( \delta: \begin{cases} 1, \text{endpoint} \\ 0, \text{otherwise} \end{cases} \)
\( z_t: \text{waypoint height} \)
\( h_t: \text{TIN height} \)

Minimize flight time

Minimize height above terrain
Block Diagram of the Solving Process

Pre-processing: (Matlab) Delaunay Triangulation of iso-probable collision surface

- Iso-probable collision surface
- Coordinates of start and finish
- Boundaries for waypoint coordinates
- Vertex coordinates and set of triangles a vertex belongs to
- Helicopter parameters: speeds and curvature
- Additional parameters

Building the model: (PuLP)
- Define variables
- Formulate constraints
- Set objective function

Solving the model: (Cplex)

Coordinates of the waypoints (trajectory)
Case Study – Oberengadin

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MILP – Resulting Trajectory

Trajectory on TIN

Start

Finish

North

East

Start

Finish

Height

North

East

TIN-Terrain

Start point

Finish point

Waypoints

Trajectory

TIN-Terrain

Start point

Finish point

Trajectory

Trajectory on TIN

Start

Finish
Case Study – Oberengadin

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Effect of Triangulation Error

TIN-Error: 100m
Triangles: 20

Start

TIN-Error: 50m
Triangles: 46

Start

52s

56s
Effect of Horizontal Curvature Constraint

TIN-Error: 100m
Without curvature constraint: 52s

TIN-Error: 100m
With curvature constraint: 52s
TIN $\rightarrow$ Iso-probable Surface $\rightarrow$ DTM

Trajectory on Terrain

Finish
Trajectory Validation

Iso-probable surface

- Uncertainty in position

Time-to-go approach (by Roxane Pott (2019))

- Uncertainty in position and velocity
- $\tau = \frac{d}{v}$
Validation of the Optimized Trajectory

Collision probabilities based on time-to-go until collision with terrain

\[ \text{time-to-collision} = \tau = \frac{d}{v} \]

Probability of collision = \( P(\tau < 40 \text{ s}) \)

Monte Carlo Simulation

R. Pott (2019)
Trajectory Validation

Trajectory with Probability for Collision

Start

Finish

Probability of collision < 40s

North

East

Height

2500

2000

785000

788000

1

0.8

0.6

0.4

0.2

0
Trajectory Validation

Collision Probabilities for waypoint 26

Collision Probabilities for waypoint 46

Institute for Geodesy and Photogrammetry (IGP)
Mathematical and Physical Geodesy (MPG)
Case Study – Oberengadin

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Limitations – Longer Trajectories

Trajectory on Terrain

Start

Finish

North

East

Height

770000

778000

786000

148000

158000

2000

3000
Limitations – Triangulation

Trajectory on Terrain

Start

Finish

Height

East

North
Conclusion & Outlook

- Mixed Integer Linear Programming
  - Linear concepts
  - Yields optimal solution

- Computationally expensive
  - Lots of variables and constraints

- Model improvements
  - Adjust triangulation
  - Include take-off & landing

• suitable
• requires approximations

• model reductions required
• heuristic approach

• to meet safety level
• to provide full trajectories